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SOLUTION OF A PROBLEM.

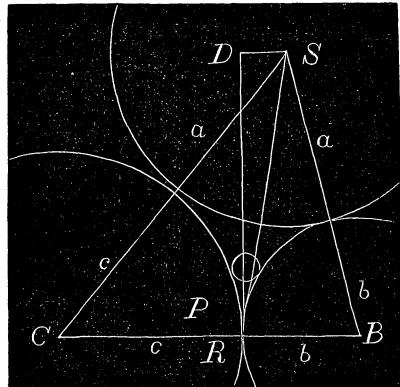
BY ISAAC H. TURRELL, CUMMINSVILLE, OHIO.

THREE circles, radii a, b, c , touch each other externally; required the radii, a_1, b_1, c_1 of three circles described in the space enclosed by them, each touching the other two and two of the given circles.

A solution of this problem can be easily obtained by means of the equation proved at page 24, Vol. II. of the ANALYST.

$T^2 = 8aa_1$, T being the direct common tangent of the circles whose radii are a, a_1 and centers S, S' ; this being a pair that do not touch each other.

Since the circles S, S' , touch B, C , externally, their center of similitude P , which is the intersection of their direct common tangents, will be in the radical axis DR . If N, N' be the points where this common tangent, which is not shown in the figure, touches S, S' , it is well known that $PR^2 = PN \cdot PN'$. Again,

$$\frac{a}{a_1} = \frac{PN}{PN'} \frac{a-a_1}{a_1} = \frac{PN-PN'}{PN'} = \frac{T}{PN'} = \frac{2\sqrt{(2aa_1)}}{PN'}, \therefore PN = \frac{2a_1\sqrt{(2aa_1)}}{a-a_1}.$$


$$\text{Similarly } PN = \frac{2a\sqrt{(2aa_1)}}{a-a_1}. \therefore PR = \frac{2aa_1\sqrt{2}}{a-a_1}.$$

Draw SD perpendicular to the radical axis DR ; then DR being the altitude of the triangle CBS , its value is $\sqrt{[abc(a+b+c)]} \div \frac{1}{2}(b+c)$.

$$\text{Also } (a+b)^2 - (b-SD)^2 = DR^2 = (a+c)^2 - (c+SD)^2 \\ \therefore SD = a(c-b) \div (c+b)$$

$$\text{Again } \sqrt{(PN^2 + a^2 - SD^2)} = PD = DR - PR; \text{ whence}$$

$$\sqrt{\left[\frac{8a^3a_1}{(a-a_1)^2} + a^2 - \frac{a^2(c-b)^2}{(c+b)^2} \right]} = \frac{2\sqrt{[abc(a+b+c)]}}{b+c} - \frac{2aa_1\sqrt{2}}{a-a_1},$$

an equation of the second degree in a_1 .

Squaring this equation, reducing, and solving with reference to a_1 ,

$$a_1 = \frac{a \{ bc+ac+ab+\sqrt{[2abc(a+b+c)]} \} \pm a^2(b+c) \pm a \sqrt{[2abc(a+b+c)]}}{bc+2ab+2ac+2\sqrt{[2abc(a+b+c)]}}$$

$$\text{whence } a_1 = a \text{ or } \frac{1}{1 \div m - 1 \div a}, \text{ where } \frac{1}{m} = \frac{2}{a} + \frac{2}{b} + \frac{2}{c} + 2\left(\frac{2}{ab} + \frac{2}{bc} + \frac{2}{ca}\right)^{\frac{1}{2}}$$

$$\text{Similarly } b_1 = \frac{1}{1 \div m - 1 \div b}, c_1 = \frac{1}{1 \div m - 1 \div c}, \text{ the radii required.}$$

The first value of a_1 is worthy of note; for if it were required to determine the position of four circles b, c, b_1, c_1 of given magnitude, that touch each other consecutively, so that a circle could be drawn touching all, the equation $a_1 = a$, shows that each of these circles fulfills this condition.

Again, a, b, c , being the radii of the first, we have the following relations connecting the radii of the circles of the various groups.

$$\left. \begin{array}{l} \frac{1}{a_1} = \frac{1}{m} - \frac{1}{a} \\ \frac{1}{b_1} = \frac{1}{m} - \frac{1}{b} \\ \frac{1}{c_1} = \frac{1}{m} - \frac{1}{c} \end{array} \right\} \text{2nd, } \left. \begin{array}{l} \frac{1}{a_2} = \frac{1}{m_1} - \frac{1}{a_1} \\ \frac{1}{b_2} = \frac{1}{m_1} - \frac{1}{b_1} \\ \frac{1}{c_2} = \frac{1}{m_1} - \frac{1}{c_1} \end{array} \right\} \text{3rd, } \left. \begin{array}{l} \frac{1}{a_x} = \frac{1}{m_{x-1}} - \frac{1}{a_{x-1}} \\ \frac{1}{b_x} = \frac{1}{m_{x-1}} - \frac{1}{b_{x-1}} \\ \frac{1}{c_x} = \frac{1}{m_{x-1}} - \frac{1}{c_{x-1}} \end{array} \right\} x+1.$$

Now substituting for $1 \div a_1, 1 \div b_1, 1 \div c_1$ in the third group, their values as given in the second, and carrying the resulting values of $1 \div a_2, 1 \div b_2, 1 \div c_2$ into the fourth, and so on, to the $(x+1)$ th group, we find that

$$\frac{1}{a_x} + \frac{1}{a} = \frac{1}{b_x} + \frac{1}{b} = \frac{1}{c_x} + \frac{1}{c} = \frac{1}{m_{x-1}} - \frac{1}{m_{x-2}} + \dots + \frac{1}{m},$$

when x is odd, and

$$\frac{1}{a_x} - \frac{1}{a} = \frac{1}{b_x} - \frac{1}{b} = \frac{1}{c_x} - \frac{1}{c} = \frac{1}{m_{x-1}} - \frac{1}{m_{x-2}} + \dots - \frac{1}{m}$$

when x is an even number.

A PROBLEM IN SURVEYING.

BY T. J. LOWRY, M. S., SAN FRANCISCO, CALIFORNIA.

Problem:—Required the positions of the two places of observation y and m , with reference to three known points A, B and C , having observed at m the angles AmB and Bmy and at y the angles ByA and Cym .

Trig. Analysis:—In the isosceles $\triangle ABe$ we have the base AB and $\angle AeB$ ($= 2AmB$), and hence all the \angle s to find Ae or Be . And in $\triangle Ade$ are known $\angle Aed$ ($= 180^\circ - AmB$), side Ae , and $\angle Ade$ ($= AyB$) to get Ad and de . Now in isosceles $\triangle Age$ having sides Ae and ge , and $\angle Aeg$ [$=$

